KEMENTERIAN PENDIDIKAN TINGGI
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI

BASIC ALGEBRA

LECTURE NOTES & WORKSHEET

Authors:
NURUL AMALINA BINTI IBRAHIM
MOHD NAWI BIN AB RAHMAN @ ISMAIL

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NURUL AMALINA BINTI IBRAHIM

MOHD NAWI BIN AB RAHMAN @ ISMAIL

DEPARTMENT OF MATHEMATICS, SCIENCE AND COMPUTER

POLITEKNIK JELI KELANTAN

2024

BASIC ALGEBRA

Published by:
Politeknik Jeli Kelantan
Jalan Raya Timur Barat
17600 Jeli
Kelantan

Email:

amalina@pjk.edu.my

nawi@pkb.edu.my

First Edition 2024

Cataloguing-in-Publication Data

Perpustakaan Negara Malaysia

A catalogue record for this book is available from the National Library of Malaysia

eISBN 978-967-2760-23-8

PREFACE

AUTHORS

NURUL AMALINA BINTI IBRAHIM

amalina@pjk.edu.my

MOHD NAWI BIN AB RAHMAN @ ISMAIL

nawi@pkb.edu.my

Thankful to Allah S.W.T for His Bless and give us strength, health, patience and motivation along the process of completing this e-book.

Our extended appreciation to the supportive Head of Department and special thanks to all colleagues of Mathematics, Science and Computer Department for their support and making this impossible process possible.

As an initiative, this e-book was written to assist in Teaching and Learning process as it provides step by step example followed by exercise to enhance students as well as helps students to understand more about Basic Algebra.

Thus, we are really hope that this e-book will be useful for students in enhancing their Mathematics skills and developing their interest towards Mathematics course. And hopefully also will be additional reference for lecturers during T&L process.

Sincerely,

Authors

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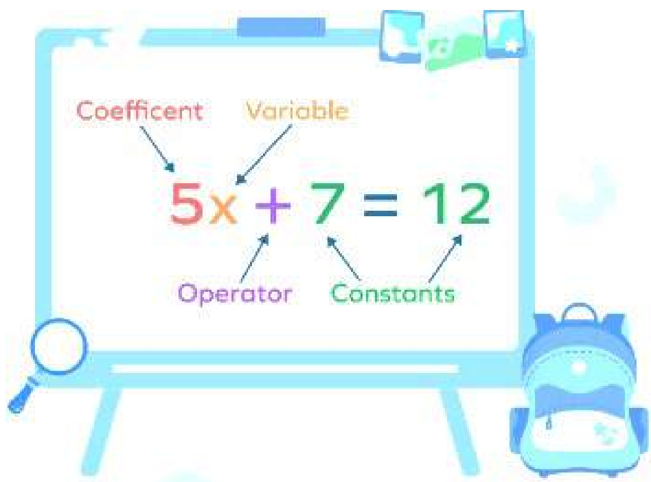
BASIC ALGEBRA

This topic cover simplifies basic algebra (solve algebraic expressions, factorization and expansion of algebraic equations and solving algebraic fractions), show algebraic equation as unknown to be subject formula and solving the simultaneous linear equation with two variables.

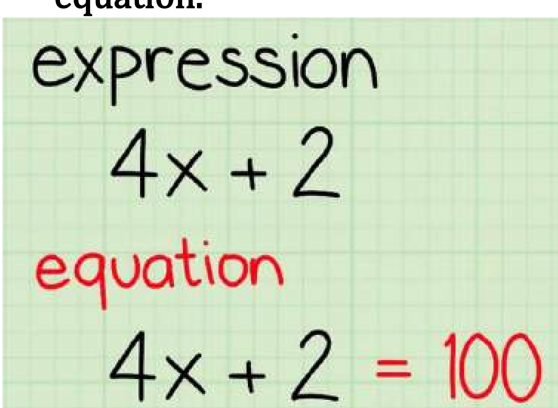


1.0 INTRODUCTION TO BASIC ALGEBRA

1.1 HOW TO SOLVE ALGEBRAIC EXPRESSIONS

PART 1 : UNDERSTANDING THE BASIC

Understand the difference between an algebraic expression and algebraic equation.



a. Algebraic Expression : a mathematical phrase that contain number/variables. It ***does not contain an equal sign and cannot be solved***

b. Algebraic Equation : ***can be solved*** and does include a series of algebraic

PART 2 : KNOW HOW TO COMBINE LIKE TERMS

Combining like terms just mean adding up (or subtracting) the terms of the same degree. This mean x^2 combined with x^2, x^3 combined with x^3 and numbers (such as 8 or 5) can be combined.

$$= 3x^2 + 5 + 4x^3 - x^2 + 2x^3 + 9$$
$$= 3x^2 - x^2 + 4x^3 + 2x^3 + 5 + 9$$
$$= 2x^2 + 6x^3 + 14$$

PART 3 : KNOW HOW TO FACTOR A NUMBER

$$3x + 15 = 9x + 30$$
$$\frac{3x + 15}{3} = \frac{9x + 30}{3}$$
$$\frac{3x}{3} + \frac{15}{3} = \frac{9x}{3} + \frac{30}{3}$$
$$x + 5 = 3x + 10$$

With an algebraic equation, you can simplify it by factoring out a common term.
Look at the coefficients of all terms. If there is a number that can “factor out” by dividing each term by that number……then you have simplified .

PART 4 : HOW TO ISOLATE A VARIABLE

An algebraic equation, your goal is to get the variable, often known as x, on one side of the equation, while placing the constant terms on the other side of the equation.
x can isolate by division, multiplication, addition, subtraction or other operation.

$$5x + 15 = 65$$
$$\frac{5x}{5} + \frac{15}{5} = \frac{65}{5}$$
$$x + 3 = 13$$
$$x + 3 - 3 = 13 - 3$$
$$\boxed{x = 10}$$

1.1 SOLVE ALGEBRAIC EXPRESSIONS

EXAMPLE: Solve the algebraic expression below

1. $2(a-6) = -a-13$

Solution

$$2a - 12 = -a - 13$$
$$2a + a = -13 + 12$$
$$3a = -1$$
$$a = \frac{-1}{3}$$

2. $\frac{2x-3}{4} = \frac{x+1}{5}$

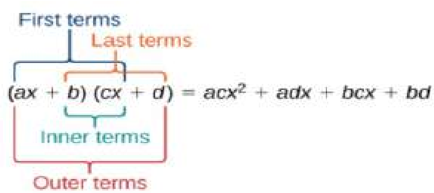
Solution

$$5(2x-3) = 4(x+1)$$
$$10x - 15 = 4x + 4$$
$$10x - 4x = 4 + 15$$
$$6x = 19$$
$$x = \frac{19}{6}$$

3. $(2x+1)(3x^2 - x + 4)$

Solution

$$= 2x(3x^2 - x + 4) + 1\,(3x^2 - x + 4)$$
$$= (6x^3 - 2x^2 + 8x) + (3x^2 - x + 4)$$
$$= 6x^3 + (-2x^2 + 3x^2) + (8x - x) + 4$$
$$= 6x^3 + x^2\ 7x + 4$$



4. $xy + 3yx - 4y^2x$

Solution

$$= (xy + 3yx) - 4y^2x$$
$$= 4xy - 4y^2x$$

"CLICK THE BUTTON ABOVE FOR MORE DETAILED

EXERCISE 1.0

i. Solve the algebraic equation below $3a + 4ba - 2a^2 + 4a^2 + 6a$	ii. Solve the algebraic equation below $a^{-1} + 2a^{-2} + 3a^{-2}$
iii. Solve the algebraic equation below $-2a^2bc^2 - (-6a^2bc^2)$	iv. Solve the algebraic equation below $-\frac{2y}{3}(9y - 3z + 6x)$
v. Solve the algebraic equation below $8x + 4y - y + 3x + z$	vi. Solve the algebraic equation below $3x^2 + 5 + 4x^3 - x^2 + 2x^3 + 9$

vii. Solve the algebraic equation below $2x^2(4xy-5)-8yx^3+9x$	viii. Solve the algebraic equation below $3a\,(2b-3c+4d)-2a\,(3b-c+6d)$
ix. Solve the algebraic equation below $6+4(3-x)$	x. Solve the algebraic equation below $2(3x+1)-(2x-3)$
xi. Solve the algebraic equation below $-x(x^2+4x-3)$	xii. Solve the algebraic equation below $2(3p+2)-3(2p-3)$

CHECK YOUR ANSWER HERE

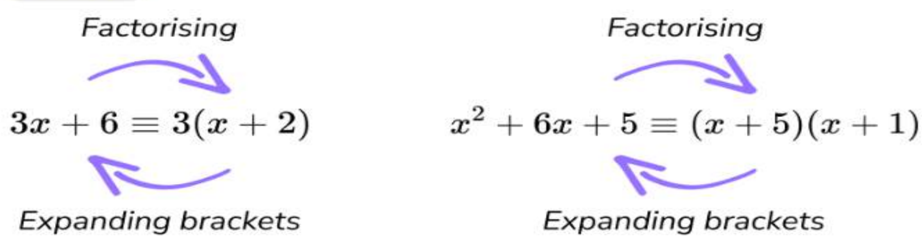
1.2 FACTORIZATION OF ALGEBRAIC EXPRESSIONS

PART 1 : UNDERSTANDING FACTORIZATION OF ALGEBRAIC EXPRESSIONS

Factorising

Factorising is the reverse process of expanding brackets.
To factorise an algebraic expression means to put it into brackets by taking out the common factors.

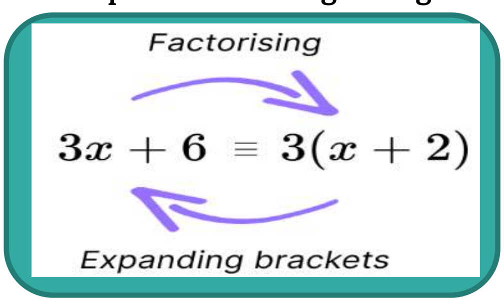
Examples



PART 2 : HOW TO FACTORISE EXPRESSIONS

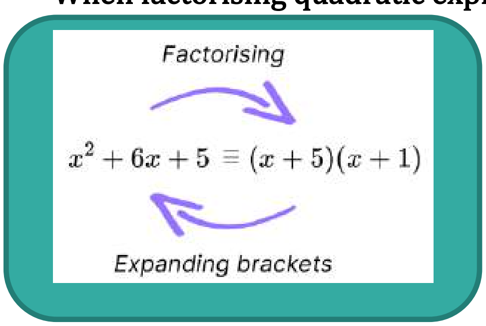
a. Factorising single bracket

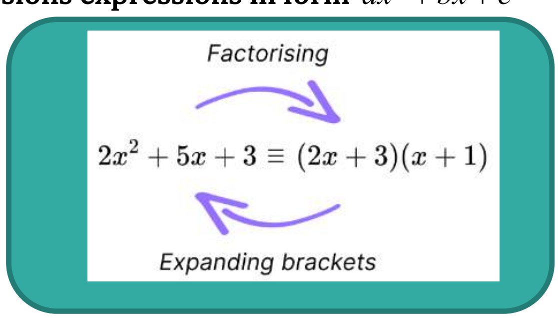
Example of factorising an algebraic expression



b. Factorising double brackets

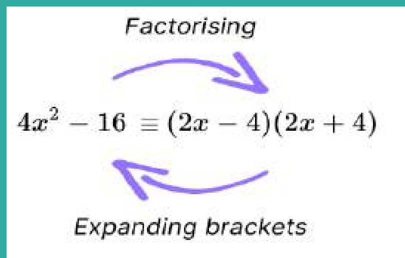
When factorising quadratic expressions expressions in form $ax^2 + bx + c$





c. Differences of two squares

Using the difference of two squares



PART 3 : FACTORISATION METHODS

Each method of factorising or factoring expressions is summarised below.

1. Factorising single brackets (example : $3x+6$)
   a. Find the highest common factor (HCF) of the numbers *3* (the coefficient of x) and *6* (the constant).
   The highest common factor (HCF) of $3x$ and *6* is *3*
   b. Write the highest common factor (HCF) at the front of the single bracket. $3(\quad + \quad)$
   c. Fill in each term in the bracket by multiplying out. $3(x+2)$

2. a) Factorising quadratics into double brackets : $x^2 + 6x + 5$
   i. Write out the factor pairs of the last numbers *(5); 1 , 5*
   ii. Find a pair of factor that + to give the middle number (6) and x to give the last number (5) : $1 \times 5 = 5$; $1 + 5 = 6$
   iii. Write two bracket and put the variable at the start of each one $(x \quad)(x \quad)$
   iv. Write one factor in the first bracket and the other factor in the second bracket. The order isn't important, the signs of the factor are. $(x+1)(x+5)$

2 b) Factorising quadratics into double brackets : $2x^2 + 5x + 3$

i. Multiply the end numbers together (2 and 3) then write out the factor pairs of this new number in order *(6); 1 , 5 ,2,3*

ii. We need a pair of factor that + to give the middle middle number (5) and x to give this new number (6) : $2+3=5$; $2 \times 3 = 6$

iii. Rewrite the original expression , this time splitting the middle term into the two factor we found in step (ii) $2x^2 + 2x + 3x + 3$

iv. Split the equation down the middle and factorise fully each half. $2x(x+1)+3(x+1)$

v. Factorise the whole expression by bringing whatever is in the bracket to the front and writing the two other terms in the other bracket. $(2x+3)(x+1)$

3. Difference of two squares : $4x^2 - 9$

a. Write down 2 bracket $(\quad)(\quad)$

b. Square root the first term and write it on the left-hand side of both bracket $\sqrt{4x^2} = 2x$; $(2x \quad)(2x \quad)$

c. Square root the last term and write it on the right hand side of both bracket. $\sqrt{9} = \pm 3$; $(2x \quad 3)(2x \quad 3)$

d. Put + in the middle of one bracket and – in the middle of the other $(2x+3)(2x-3)$

FACTORIZATION OF ALGEBRAIC EQUATIONS

EXAMPLE : Solve the following algebraic equation

1. $p^2 + 6p - 16$

Solution

$= (p-2)(p+8)$

2. $x^2 - 8x = -15$

Solution

$$x^2 - 8x + 15 = 0$$
$$(x-3)(x-5) = 0$$
$$(x-3) = 0 \quad ; \quad (x-5) = 0$$
$$x = 3 \quad ; \quad x = 5$$

3. $5x^2 + 7x - 9 = 4x^2 + x - 18$

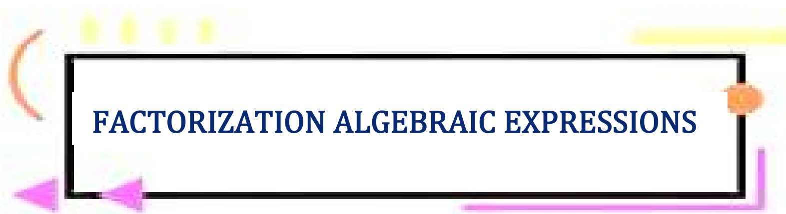
Solution

$$5x^2 + 7x - 9 - 4x^2 - x + 18 = 0$$
$$(5x^2 - 4x^2) + (7x - x) + (-9 + 18) = 0$$
$$x^2 + 6x + 9 = 0$$
$$(x+3)(x+3) = 0$$
$$x + 3 = 0$$
$$x = -3$$

4. $ax - ay + 2x - 2y$

Solution

$$= a(x-y) + 2(x-y)$$
$$= (x-y)(a+2)$$



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EXERCISE 2.0

i. Solve the algebraic equation. $x^2 - 5x - 10 = -4$	ii. Solve the algebraic equation. $3 - x - 2x^2 = 0$
iii. Solve the algebraic equation. $\frac{2x+7}{3x-2} = x$	iv. Solve the algebraic equation. $x^2 - 2x = 15$
v. Solve the algebraic equation. $p^2 = -10p - 9$	vi. Solve the algebraic equation. $m^2 - 7m + 16 = 4$

vii. Solve the algebraic equation.

$$3x^2 - 2x(x-3) + 9$$

viii. Solve the algebraic equation.

$$x^2 + 4x - 9 = 2(x-3)$$

ix. Solve the algebraic equation.

$$\frac{3x(x-1)}{2} = x + 6$$

x. Solve the algebraic equation.

$$4x^2 - 15 = 17x$$

xi. Solve the following algebraic eqution

$$5x^2 + 4x = 3(2-x)$$

xii. Solve the following algebraic eqution

$$x - 1 = \frac{6-3x}{2x}$$

CHECK YOUR ANSWER HERE

1.3 EXPANSION OF ALGEBRAIC EXPRESSIONS

PART 1 : UNDERSTANDING EXPANSION OF ALGEBRAIC EXPRESSIONS

Expand and Simplify

Expand and simplify combines two algebraic processes:

- **Expanding brackets**, where we remove brackets by multiplying them out;
- **Simplifying** the expression by **collecting like terms**.

Example Expand and simplify $2(x+5)+3(x-2)$

1. Expand the brackets $2(x+5)+3(x-2)=2x+10+3x-6$
2. Collect like terms $2x+10+3x-6=5x+4$

The simplified expression is $5x+4$

PART 2 : HOW TO EXPAND AND SIMPLIFY BRACKET

In order to expand and simplify brackets :

i. Expand each bracket in the expression

ii. Collect the like terms

PART 3 : EXPLAIN HOW TO EXPAND AND SIMPLIFY

a. Expand and simplify with two or more bracket

$(x+5)(x-1)$

$=x^2-5x-x-5$

$=x^2+4x-5$

b. Expand and simplify with two or more bracket

$(x+5)(x-1)$

$=x^2-5x-x-5$

$=x^2+4x-5$

EXPANSION OF ALGEBRAIC EQUATIONS

EXAMPLE: Simplify each of the following

1. $(x+2)(x+3)$

Solution

$$=x(x+3)+2y(x+3)$$
$$=x^2+3x+2xy+6y$$

$(a+b)(c+d)=a(c+d)+b(c+d)$
$=ac+ad+bc+cd$

2. $(2x+y)(x-2y)$

Solution

$$=2x(x-2y)+y(x-2y)$$
$$=2x^2-4xy+xy-2y^2$$
$$=2x^2-3xy-2y^2$$

$*(a+b)(c-d)=a(c-d)+b(c-d)$
$=ac-ad+bc-bd$

3. $(2x+1)^2$

Solution

$$=(2x+1)(2x+1)$$
$$=2x(2x+1)+1(2x+1)$$
$$=4x^2+2x+2x+1$$
$$=4x^2+4x+1$$

$*(a+b)^2=a^2+2ab+b^2$

4. $(x-2y)^2$

Solution

$$=(x-2y)(x-2y)$$
$$=x(x-2y)-2y(x-2y)$$
$$=x^2-2xy-2xy+4y^2$$
$$=x^2-4xy+4y^2$$

$*(a-b)^2=a^2-2ab+b^2$

EXPAND ALGEBRAIC EXPRESSIONS

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EXERCISE 3.0

i. Simplify the algebraic expression below $(x+2)\,(3x^2+2x-1)$	ii. Simplify the algebrraic expressions $-7x(2x-5)$
iii. Simplify the algebraic expression below $(4x^2+\frac{3}{2}y^2)^2$	iv. Simplify the algebraic expression below $2(x+1)-3(4-2x)$
v. Solve the algebraic equation below $2(x+1)=3(4-2x)$	vi. Solve the algebraic equation below $(x+1)^2=4(x+4)$

vii. Solve the algebraic expressions below $2x(3x-4y)-(x-y)\ (x+3y)$	viii. Solve the algebraic expressions below $(2y-3x)(x-4y)$
ix. Solve the algebraic expressions below $(p-q)(p+q)+p(p-q)$	x. Solve the algebraic expressions below $2(x-3y)^2+xy$

CHECK YOUR ANSWER HERE

1.4 SOLVING ALGEBRAIC FRACTIONS

PART 1 : UNDERSTANDING SOLVING ALGEBRAIC FRACTIONS

Algebraic Fractions

Algebraic fractions are fractions that contain at least one variable.

Examples

$\frac{x}{12}$ — x is the numerator	$\frac{3}{x+1}$ — The denominator is an expression in terms of x	$\frac{2x}{15}$ — The numerator is a multiple of x
$\frac{x+1}{2x}$ — Both the numerator and the denominator contain an x term	$\frac{3x+4}{2x-5}$ — Both the numerator and the denominator contain an expression with x	$\frac{(3x+4)^2}{x^2-9}$ — The numerator and the denominator are quadratic expressions

PART 1 : HOW TO SOLVE EQUATIONS INCLUDING ALGEBRAIC FRACTION

In order to seolve equation including algebraic fraction:

1. Convert each fraction so they all have a common denominator
2. Multiply the equation throughout by the common denominator
3. Solve the equation(linear or quadratic)

PART 2 : ALGEBRAIC FRACTION EXAMPLES

a. Equation with one fraction ; $\frac{2x-1}{3}+x=3$

i. Convert each fraction so they all have a common denominator. *(here we only have one fraction so not need to convert)*

ii. Multiply the equation throughout by the common denominator .*(multiply the equation throughout by 3 (the denominator)*:

$$\frac{2x-1}{3_{\times 3}}+x=3_{\times 3}$$

$$=2x-1+3x=9$$

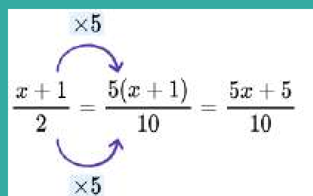
Make sure that you multiply every term in the equation by 3

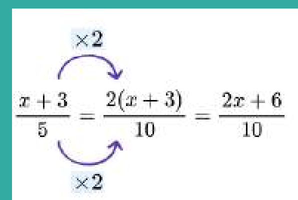
iii. Solve the equation (linear or quadratic)

$$2x-1+3x=9$$
$$5x-1=9$$
$$5x=10$$
$$x=2$$

b. Equation with two fraction ; $\frac{x+1}{2}+\frac{x+3}{5}=6$

i. Convert each fraction so they all have a common denominator. *(here we only have two fraction with denominators of 2 and 5, the lowest common multiple of 2 and 5 is 10 and so can convert to the same denominator)*





We now have the equation : $\frac{5x+5}{10}+\frac{2x+6}{10}=6$

ii. Multiply the equation through out by the common denominator .*(multiply the equation throughout by 10 (the denominator)*:

$$\frac{5x+5}{10}+\frac{2x+6}{10}=6$$
$$5x+5+2x+6=60$$

Make sure that you multiply every term in the equation by 10

iii. Solve the equation (linear or quadratic)

$5x+5+2x+6=60$

$7x+11=60$

$7x=49$

$x=7$

c. Equation with x fraction ; $3-\frac{5}{x+1}=1$

i. Convert each fraction so they all have a common denominator. *(here we only have one fraction so no need find common denominator)*

ii. Multiply the equation through out by $x+1$ (the common denominator .)

$$(x+1)\times 3-\frac{5}{x+1}=1\times(x+1)$$
$$3(x+1)-5=1(x+1)$$

iii. Solve the equation (linear or quadratic)

$$3(x+1)=1(x+1)$$
$$3x+3-5=x+1$$
$$3x-2=x+1$$
$$2x-2=1$$
$$2x=3$$
$$x=\frac{3}{2}$$

d. Equation with three fraction ; $\frac{1}{x}+\frac{1}{2x}+\frac{1}{3x}=11$

i. Convert each fraction so they all have a common denominator. *(need to find the lowest common multiple of x,2x and 3x. We can convert each fraction to have the common denominator of 6x)*

$$\frac{1}{x}=\frac{6}{6x}\ (\times 6) \qquad \frac{1}{2x}=\frac{3}{6x}\ (\times 3) \qquad \frac{1}{3x}=\frac{2}{6x}\ (\times 2)$$

So we have an equation:

$$\frac{6}{6x}+\frac{3}{6x}+\frac{2}{6x}=11$$

$$\frac{6+3+2}{6x}=11$$

$$\frac{11}{6x}=11$$

ii. Multiply the equation through out by $6x$ (the common denominator .)

$$6x\times\frac{11}{6x}=11\times 6x$$

$$11=6x$$

iii. Solve the equation (linear or quadratic)

$$11=6x$$

$$\frac{1}{6}=x$$

FACTORIZATION ALGEBRAIC EXPRESSIONS

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SOLVING ALGEBRAIC FRACTIONS

EXAMPLE : Simplify each of the following

1. $\frac{2x+2}{3}-\frac{x+5}{3}$

Solution

$=\frac{2x+2-x-5}{3}$

$=\frac{x-3}{3}$

** Same denominators*

2. $\frac{2x}{7}+\frac{x}{14}$

Solution

$=\frac{2x\times 2}{7\times 2}+\frac{x}{14}$

$=\frac{4x+x}{14}$

$=\frac{5x}{14}$

**Equalise the denominators*

3. $\frac{s}{3t}-\frac{s}{5}$

Solution

$=\frac{s\times 5}{3t\times 5}-\frac{s\times 3t}{5\times 3t}$

$=\frac{5s-3st}{15t}$

** No common factor. Multiply both denominators to get a common denominators*

4. $\frac{3}{10x}+\frac{2}{15x}$

Solution

$=\frac{3\times 3}{10x\times 3}+\frac{2\times 2}{15x\times 2}$

$=\frac{9}{30x}+\frac{4}{30x}$

$=\frac{13}{30x}$

5. $\frac{3}{x+2}\times\frac{2x+4}{9x}$ — *factorise*

Solution

$=\frac{3}{x+2}\times\frac{2(x+2)}{9x}$

$=\frac{{}^{1}\cancel{3}}{\cancel{x+2}}\times\frac{2(\cancel{x+2})}{{}_{3}\cancel{9}x}$ → *simplify*

$=\frac{2}{3x}$

6. $\frac{15x}{4x-8}\div\frac{3x}{(x-2)^2}$ — *Change* $\div\rightarrow\times$

Solution

$=\frac{15x}{4x-8}\times\frac{(x-2)^2}{3x}$

$=\frac{{}^{5}\cancel{15x}}{4\cancel{(x-2)}}\times\frac{\cancel{(x-2)}(x-2)}{\cancel{3x}_{1}}$ → *simplify*

$=\frac{5x-10}{4}$

EXERCISE 4.0

i. Simplify each of the algebraic expression $\frac{2h+3}{7k^2}-\frac{h-4}{7k^2}$	ii. Simplify each of the algebraic expression $\frac{2}{p+q}+\frac{5r^2}{4r}$
iii. Simplify each of the algebraic expression $\frac{2a}{3}-\frac{3a}{5}$	iv. Simplify each of the algebraic expression $\frac{1}{6n}+\frac{n}{3m^2}$

v. Simplify each of the algebraic expression $$\frac{x}{3}-\frac{x^2-4}{6x}$$	vi. Simplify each of the algebraic expression $$\frac{x}{3}-\frac{x+2}{5}$$
vii. Simplify each of the algebraic expression $$\frac{3b-a}{4a-10b}\div\frac{5a^2-15ab}{6a^2-15ab}$$	viii. Simplify each of the algebraic expression $$\frac{p^2-q^2}{3p-q}\times\frac{9p-3q}{(p+q)^2}$$

ix. Simplify each of the algebraic expression $\dfrac{4x}{x-2y} \div \dfrac{10xy}{4x-8y}$	x. Simplify each of the algebraic expression $\dfrac{a^2-9}{3ab} \times \dfrac{6b^2}{a+3}$
xi. Simplify each of the algebraic expression $\dfrac{(p+q)^2}{12p+9q} \div \dfrac{p^2-q^2}{8p+6q}$	xii. Simplify each of the algebraic expression $\dfrac{14ab^2 \times 3dc^2}{35a^3b^3}$

CHECK YOUR ANSWER HERE

2.0 SHOW ALGEBRAIC EQUATION AS UNKNOWN TO BE SUBJECT FORMULA

Rearranging equations changes the form of the equation to display it in a different way. This is sometimes called changing the subject.

Steps to rearrange the equation:

01	Identify the variable that need to make the subject
02	Move other variables and coefficients (constants /numbers) to the other side of the equation to leave variable that need by: - Removing any fractions by multiplying by the denominators - Adding or subtracting terms near to the variable - Dividing by the coefficient of the variable - Taking a root or power of both sides of the equation
03	Rearrange the equation so each term containing the term that want to be a subject is on one side of the equation (normally on Left-hand side).
04	Factorization may be needed if there are multiple terms the subject
05	Perform an operation to ensure only a single variable is left as the subject

EXAMPLE 2.0:

Convert the following equations so that the variable in the bracket will be the subject of formula. Write the answer in the simplest form.

1. $2x-5y=10$ $[x]$

Solution:

$$2x=10+5y$$
$$x=\frac{10+5y}{2}$$
$$x=\frac{5(2+y)}{2}$$

2. $y=\dfrac{x+3}{x-8}$ $[x]$

Solution:

$$y(x-8)=x+3$$
$$xy-8y=x+3$$
$$xy-x=3+8y$$
$$x(y-1)=3+8y$$
$$x=\frac{3+8y}{y-1}$$

3. $V=\dfrac{1}{3}\pi r^2 h$ $[r]$

Solution:

$$\frac{1}{3}\pi r^2 h=V$$
$$\pi r^2 h=3V$$
$$r^2=\frac{3V}{\pi h}$$
$$r=\sqrt{\frac{3V}{\pi h}}$$

4. $x = 2 + \sqrt{\dfrac{x}{y}}$ $[y]$

Solution:

$$\sqrt{\frac{x}{y}} = x - 2$$

$$\frac{x}{y} = (x-2)^2$$

$$x = y \times (x-2)^2$$

$$y = \frac{x}{(x-2)^2}$$

5. $s = vt - \dfrac{1}{2}at^2$ $[a]$

Solution:

$$\frac{1}{2}at^2 = vt - s$$

$$at^2 = 2(vt - s)$$

$$a = \frac{2(vt - s)}{t^2}$$

EXERCISE 5.0:
Convert the following equations so that the variable in the bracket will be the subject of formula. Write the answer in the simplest form.

1. $v = u + at$ $[a]$	2. $4(x-y) = 5y-3$ $[y]$
3. $2x + a = b(x-2)$ $[x]$	4. $3y = \sqrt{5x} - 9$ $[x]$
5. $\frac{x}{6} - 5 = y$ $[x]$	6. $\frac{p}{q} = \frac{2x}{x+5}$ $[x]$

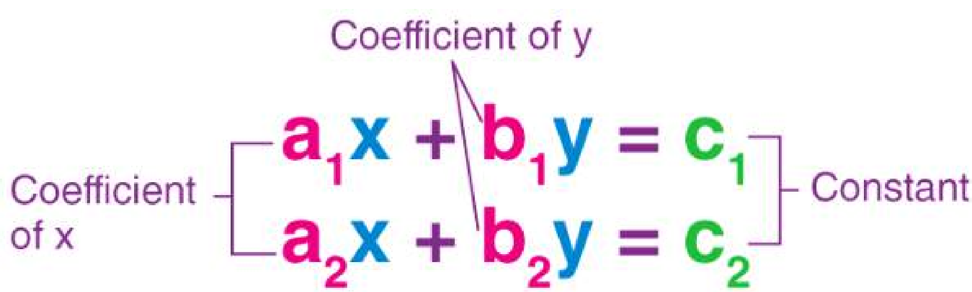
7. $p = \dfrac{3r + 7t}{tr}$ $[t]$	8. $m = \dfrac{p}{p-5}$ $[p]$
9. $h - 3 = \dfrac{\sqrt{(k+1)}}{2}$ $[k]$	10. $\sqrt{\dfrac{x+y}{x-y}} = \dfrac{1}{3}$ $[y]$

CHECK YOUR ANSWER HERE

3.0 SOLVING SIMULTANEOUS LINEAR EQUATIONS WITH TWO VARIABLES

Simultaneous linear equations are the system of two linear equations in two or three variables that are solved together to find a common solution

Simultaneous linear equation is represented as follows:



There are two commons methods to solve simultaneous linear equations:

ELIMINATION METHOD

SUBSTITUTION METHOD

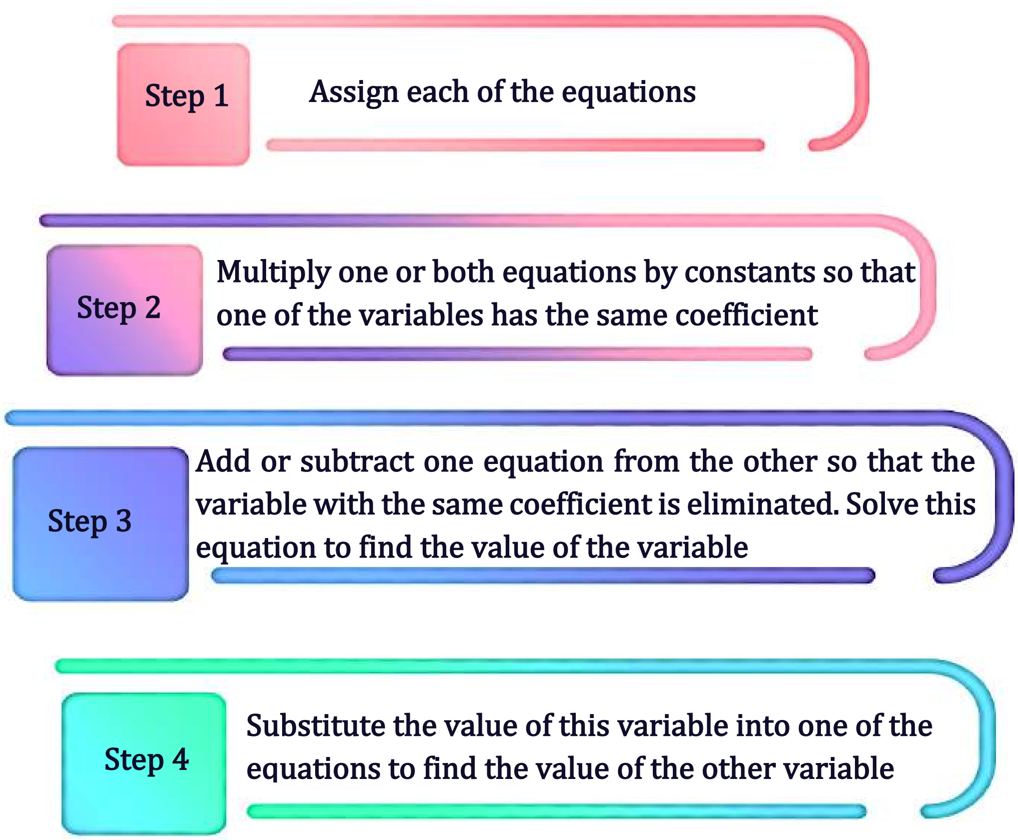
"CLICK THE BUTTON ABOVE FOR MORE DETAILED EXPLANATION"

3.1 Elimination Method

In this method, either add or subtract the equations to get the equation in one variable. If the coefficients of one of the variable are the same, and the sign of the coefficients are opposite, we can add the equation to eliminate the variable. Similarly, if the coefficients of one of the variable are the same, and the sign of the coefficients are the same, we can substract the equation to get the equation in one variable.

In case if do not have the equation to directly add or subtract the equations to eliminate the variable, we can begin by multiplying one or both the equations by a constant value on both sides of an equation to obtain the equivalent linear system of equations and then eliminate the variable by simply adding or subtracting equations.

Below is the steps involved in Elimination Method:



Example 3.1

Solve the following simultaneous equations using **Elimination Method**.

1. $5x-2y=17$

   $6x+2y=16$

Solution:

Step 1: Assign both equation

$5x-2y=17$ - - - - - Eqn (1)

$6x+2y=16$ - - - - - Eqn (2)

Step 2: Add Eqn (1) and Eqn (2) to eliminate unknown y

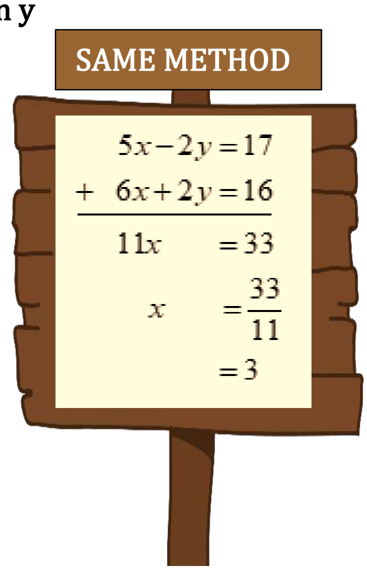
$$(5x-2y)+(6x+2y)=17+16$$

$$5x-2y+6x+2y=33$$

$$11x=33$$

$$x=\frac{33}{11}$$

$$x=3$$



Step 3: Substitute $x=3$ into Eqn (1)

$$5(3)-2y=17$$

$$15-2y=17$$

$$-2y=17-15$$

$$-2y=2$$

$$y=\frac{2}{-2}$$

$$y=-1$$

2. $5x+2y=17$
$4x+y=10$

Solution:

Step 1: Assign both equation

$5x+2y=17$ ----- Eqn (1)

$4x+y=10$ ----- Eqn (2)

Step 2: Multiply Eqn (2) by 2

$8x+2y=20$ ----- Eqn (3)

Step 3: Subtract the Eqn (1) and Eqn (3) to eliminate variable y

$$(5x+2y)-(8x+2y)=17-20$$

$$5x+2y-8x-2y=17-20$$

$$-3x=-3$$

$$x=\frac{-3}{-3}$$

$$=1$$

Step 4: Substitute $x=1$ into Eqn (1)

$$5(1)+2y=17$$

$$5+2y=17$$

$$2y=17-5$$

$$2y=12$$

$$y=\frac{12}{2}$$

$$=6$$

3. $5x-6y=28$
$4x-4y=24$

Solution:

Step 1: Assign both equation

$-2x+4y=-4$ - - - - - Eqn (1)

$5x-3y=24$ - - - - - Eqn (2)

Step 2: Multiply Eqn (1) by 5 and Eqn (2) by 2

$-10x+20y=-20$ - - - - - Eqn (3)

$10x-6y=48$ - - - - - Eqn (4)

Step 3: Adding the Eqn (3) and Eqn (4) to eliminate variable x

$$\left(-10x+20y\right)+\left(10x-6y\right)=-20+48$$

$$\begin{aligned} -10x+20y+10x-6y&=28 \\ 14y&=28 \\ y&=\frac{28}{14} \\ &=2 \end{aligned}$$

Step 4: Substitute $y=2$ into Eqn (1)

$$-2x+4\left(2\right)=-4$$

$$\begin{aligned} -2x+8&=-4 \\ -2x&=-4-8 \\ -2x&=-12 \\ x&=\frac{-12}{-2} \\ &=6 \end{aligned}$$

EXERCISE 6.0:
Solve the following simultaneous equations using **Elimination Method**.

1. $x - 3y = 1$ $2x + 5y = 35$	2. $3x + y = -1$ $2x + 4y = 10$
3. $2x + 5y = 1$ $3x - 2y = 30$	4. $5x - 6y = 28$ $4x - 4y = 24$

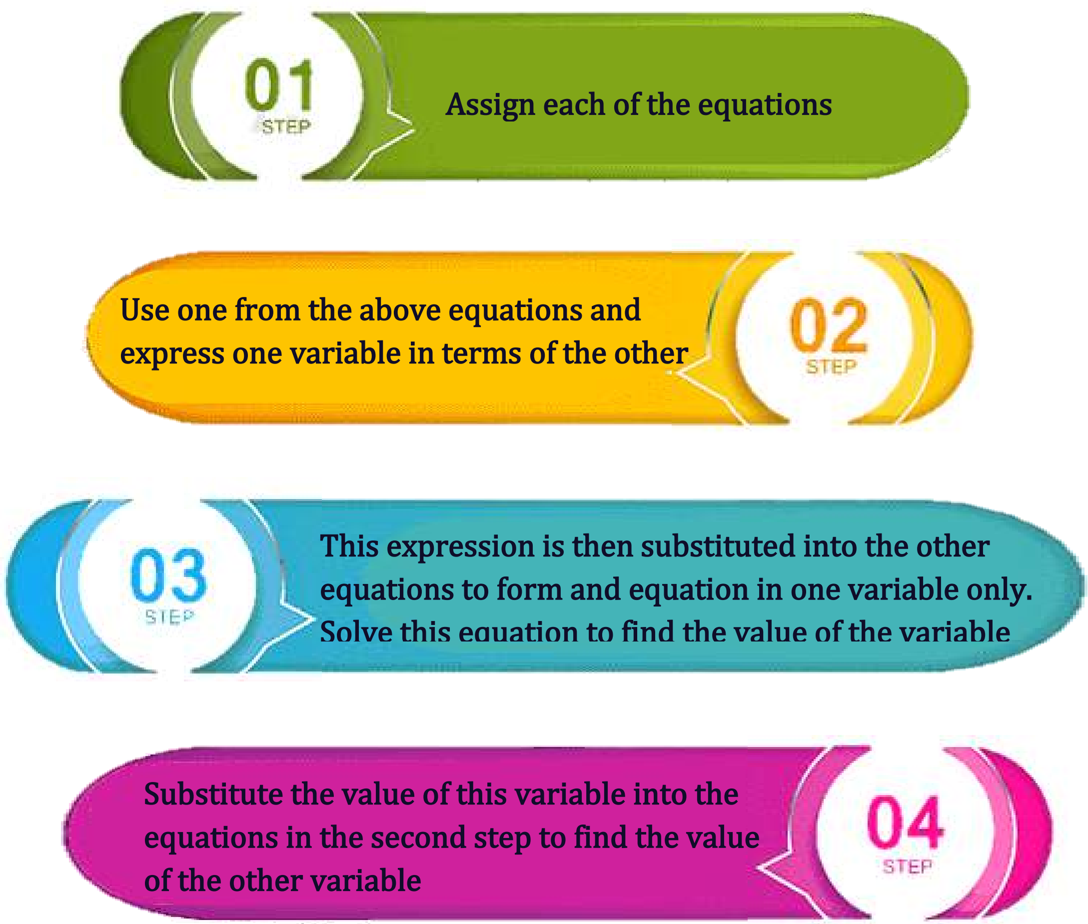
5. $6x + 2y = -2$ $4x - 3y = 29$	6. $5x - 3y = 33$ $3x - 9y = 63$

CHECK YOUR ANSWER HERE

3.2 Substitution Method

In this method, we find the value of any one of the variables by isolating it one side and taking every other term on the other side of the equation. Then substitute that value in the second equation. The substitution method is preferable when one of the variables in one of the equations has a coefficient of 1. It is involve simple steps to find the values of a variables of a system of linear equations.

In simple words, the substitution method involves substituting the value of any one of the variables from one equation into the other equation. The steps involved are:



Example 3.2

Solve the following simultaneous equations using **Substitution Method.**

1. $-2x+y=-16$
$-4x-3y=-12$

Solution:

Step 1: Assign both equation

$-2x+y=-16$ ----- Eqn (1)

$-4x-3y=-12$ ----- Eqn (2)

Step 2: From Eqn (1) make y as a subject

$y=-16+2x$ ----- Eqn (3)

Step 3: Substitute Eqn (3) into Eqn (2)

$$-4x-3(-16+2x)=-12$$

$$-4x+48-6x=-12$$

$$-4x-6x=-12-48$$

$$-10x=-60$$

$$x=\frac{-60}{-10}$$

$$=6$$

Step 4: Substitute $x=6$ into Eqn (3)

$$y=-16+2(6)$$

$$=-4$$

2. $-3x-4y=6$
$3x+6y=-12$

Solution:

Step 1: Assign both equation

$-3x-4y=6$ - - - - - Eqn (1)

$3x+6y=-12$ - - - - - Eqn (2)

Step 2: From Eqn (2) make x as a subject

$3x=-12-6y$

$x=\dfrac{-12-6y}{3}$

$x=\dfrac{-12}{3}-\dfrac{6y}{3}$

$x=-4-2y$ - - - - - Eqn (3)

Step 3: Substitute Eqn (3) into Eqn (1)

$-3(-4-2y)-4y=6$

$12+6y-4y=6$

$6y-4y=6-12$

$2y=-6$

$y=\dfrac{-6}{2}$

$=-3$

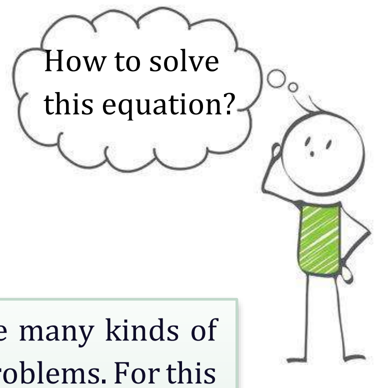
Step 4: Substitute $y=-3$ into Eqn (3)

$x=-4-2(-3)$

$=2$

3. $\frac{x}{3}+\frac{y}{5}=7$

$\frac{x}{6}-\frac{2y}{5}=-4$



In Maths there are many kinds of step to solve the problems. For this equation, we can first change the algebraic expression to linear equation or just use the algebraic expressions above.

METHOD 1

Solution:

Step 1: Assign both equation

$\frac{x}{3}+\frac{y}{5}=7$ - - - - - Eqn (1)

$\frac{x}{6}-\frac{2y}{5}=-4$ - - - - - Eqn (2)

Step 2: Eqn (1) multiply by 15 and Eqn (2) multiply by 30

$5x+3y=105$ - - - - - Eqn (3)

$5x-12y=-120$ - - - - - Eqn (4)

Step 3: From Eqn (3) make x as a subject

$5x+3y=105$

$5x=105-3y$

$x=21-\frac{3}{5}y$ - - - - - Eqn (5)

Step 4: Substitute Eqn (5) into Eqn (4)

$$5\left(21-\frac{3}{5}y\right)-12y=-120$$

$$105-3y-12y=-120$$

$$-3y-12y=-120-105$$

$$-15y=-225$$

$$y=\frac{-225}{-15}$$

$$y=15$$

Step 5: Substitute $y=15$ into Eqn (5)

$$x=21-\frac{3}{5}(15)$$

$$x=12$$

METHOD II

Solution:

Step 1: Assign both equation and write the equation as below

$$\frac{1}{3}x+\frac{1}{5}y=7 \qquad \text{- - - - - Eqn (1)}$$

$$\frac{1}{6}x-\frac{2}{5}y=-4 \qquad \text{- - - - - Eqn (2)}$$

Step 2: From Eqn (1) make x as a subject

$$\frac{1}{3}x+\frac{1}{5}y=7$$

$$\frac{1}{3}x=7-\frac{1}{5}y$$

$$x=3\left(7-\frac{1}{5}y\right)$$

$$x=21-\frac{3}{5}y \qquad \text{- - - - - Eqn (3)}$$

Step 3: Substitute Eqn (3) into Eqn (2)

$$\frac{1}{6}\left(21-\frac{3}{5}y\right)-\frac{2}{5}y=-4$$

$$\frac{7}{2}-\frac{1}{10}y-\frac{2}{5}y=-4$$

$$-\frac{1}{10}y-\frac{2}{5}y=-4-\frac{7}{2}$$

$$-\frac{1}{2}y=-\frac{15}{2}$$

$$y=\left(-\frac{15}{2}\right)\div\left(-\frac{1}{2}\right)$$

$$y=15$$

Step 4: Substitute $y=-3$ into Eqn (3)

$$x=21-\frac{3}{5}(15)$$

$$x=12$$

EXERCISE 7.0:

Solve the following simultaneous equations using **Substitution Method**.

1. $x + y = 10$ $2x + y = 17$	2. $3x + 2y = 23$ $2x - y = 6$

3. $2x-4y=2$ $3x+4y=23$	4. $4x+2y=24$ $7x+2y=33$

5. $3x + 2y = 7$ $2x + 9y = 43$	6. $3x + 4y = 11$ $7x - 3y = 1$

CHECK YOUR ANSWER HERE

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